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**Two various approaches to VTS Zatoka radar system reliability analysis**

**Keywords**  
VTS system, system reliability, shipping safety

**Abstract**  
In the paper we propose two ways of reliability calculation of radar system in Vessel Traffic Services Zatoka. Reliability and availability of the system were calculated on the base of reliability of the system components. In the first approach there was assumed that system is series, in the second approach system is treated as a series-“m out of n”. We obtain different results. Conclusion is that choosing proper method of approach to system reliability and availability analysis is decisive in appropriate evaluation of those properties.

1. Introduction

One of the most important properties of the devices and technical systems is their reliability. Reliability is extremely important when concerns systems, which assure people safety or/and natural environment protection. Vessel Traffic Services System – VTS Zatoka is that type of system. Its main task is to assure safe navigation for all ships that sails to ports of Gdynia and Gdansk. The most important part of the VTS Zatoka system are shore based maritime radars. Reliability of that system can be evaluated in different ways. In the paper there are proposed two possible approaches to calculate that reliability [1].

2. Systems’ definitions

We assume that [2]

\[ E_i, i = 1,2,...,n, n \in \mathbb{N}, \]

are two-state components of the system having reliability functions

\[ R_i(t) = P(T_i > t), t \in (-\infty, \infty), i = 1,2,...,n, \]

where

\[ T_i, i = 1,2,...,n, \]

are independent random variables representing the lifetimes of components \( E_i \) with distribution functions

\[ F_i(t) = P(T_i \leq t), t \in (-\infty, \infty), i = 1,2,...,n. \]

**Definition 1.** A two-state system is called series if its lifetime \( T \) is given by

\[ T = \min\{T_i\}. \]

![Figure 1. The scheme of a series system](image)

The above definition means that the series system is not failed if and only if all its components are not failed, and therefore its reliability function is given by

\[ R(t) = \prod_{i=1}^{n} R_i(t), t \in (-\infty, \infty). \tag{1} \]

**Definition 2.** A two-state series system is called non-homogeneous if it is composed of \( a, 1 \leq a \leq n \), different types of components and the fraction of the \( i \)th type component in the system is equal to \( q_i \), where

\[ q_i > 0, \sum_{i=1}^{a} q_i = 1. \]

Moreover

\[ R^{(i)}(t) = 1 - F^{(i)}(t), t \in (-\infty, \infty), i = 1,2,...,a, \tag{2} \]

is the reliability function of the \( i \)th type component.
The scheme of a non-homogeneous series system is given in Figure 2.

![Figure 2](image_url)

**Figure 2.** The scheme of a non-homogeneous series system

It is easy to show that the reliability function of the non-homogeneous two-state series system is given by

\[
R_{i_1\ldots i_n}(t) = \prod_{i=1}^{n} (R^{(i)}(t))^{q_i}, \quad t \in (-\infty, \infty),
\]

(3)

A two-state system is called an “m out of n” system if its lifetime \(T\) is given by

\[
T = T_{(n-m+1)}, \quad m = 1, 2, \ldots, n,
\]

where \(T_{(n-m+1)}\) is the mth maximal order statistic in the sequence of component lifetimes \(T_1, T_2, \ldots, T_n\).

The above definition means that the two-state “m out of n” system is not failed if and only if at least m out of its n components are not failed. The two-state “m out of n” system becomes a parallel system if \(m = 1\), whereas it becomes a series system if \(m = n\). The reliability function of the two-state “m out of n” system is given either by

\[
R_{(m)}^n(t) = 1 - \sum_{q_1+q_2+\ldots+q_n = m-1}^{a} \prod_{i=1}^{n} (R^{(i)}(t))^{q_i} (1-R^{(i)}(t))^{1-q_i},
\]

(4)

\(t \in (-\infty, \infty),\)

or by

\[
R_{(\bar{m})}^n(t) = \prod_{q_1+q_2+\ldots+q_n = m-1}^{a} \prod_{i=1}^{n} (R^{(i)}(t))^{q_i} (1-R^{(i)}(t))^{1-q_i},
\]

(5)

\(t \in (-\infty, \infty), \quad \bar{m} = n - m.\)

**Definition 3.** A two-state “m out of n” system is called non-homogeneous if it is composed of \(a\), \(1 \leq a \leq n\), different types of components and the fraction of the ith type component in the system is equal to \(q_i\), where \(q_i > 0, \sum_{i=1}^{a} q_i = 1.\) Moreover

\[
R^{(i)}(t) = 1 - F^{(i)}(t), \quad t \in (-\infty, \infty), \quad i = 1, 2, \ldots, a.
\]

(6)

The scheme of a non-homogeneous “m out of n” system is given in Figure 3, where \(i_1, i_2, \ldots, i_n \in \{1, 2, \ldots, n\}\) and \(i_j \neq i_k\) for \(j \neq k\).

The reliability function of the non-homogeneous two-state “m out of n” system is given either by

\[
R_{(m)}^{(n)}(t) = 1 - \sum_{q_1+q_2+\ldots+q_n = m-1}^{a} \prod_{i=1}^{n} (R^{(i)}(t))^{q_i} (1-R^{(i)}(t))^{1-q_i},
\]

(7)

\(t \in (-\infty, \infty),\)

or by

\[
R_{(\bar{m})}^{(n)}(t) = \prod_{q_1+q_2+\ldots+q_n = m-1}^{a} \prod_{i=1}^{n} (R^{(i)}(t))^{q_i} (1-R^{(i)}(t))^{1-q_i},
\]

(8)

\(t \in (-\infty, \infty), \quad \bar{m} = n - m.\)

**Figure 3.** The scheme of a non-homogeneous “m out of n” system

**Definition 4.** A multi-state system is called series- “m out of \(k_n\)” if its lifetime \(T\) is given by

\[
T = T_{(k_n-m+1)}, \quad m = 1, 2, \ldots, k_n.
\]
where $T_{(k_n-m+1)}$ is $m$-th maximal statistics in the random variables set

$$
T_i = \min \{T_j \}, \quad i=1,2,...,k_n.
$$

The above definition means that series “$m$ out of $k_n$” system is composed of $k_n$ series subsystems and it is not failed if and only if at least $m$ out of its $k_n$ series subsystems are not failed.

The reliability of the series “$m$ out of $k_n$” system is given either by

$$
R_{k_n,l_1,l_2,...,l_{k_n}}^{(m)} (t) = 1 - 
\sum_{r_1+r_2+...+r_{k_n} \leq m-1} \prod_{i=1}^{k_n} \prod_{j=1}^{l_i} [1 - R_j(t)]^{l_i} R_j(t)^{1-r_i},
$$

or by

$$
\bar{R}_{k_n,l_1,l_2,...,l_{k_n}}^{(m)} (t) =
\sum_{r_1+r_2+...+r_{k_n} \leq m} \prod_{i=1}^{k_n} \prod_{j=1}^{l_i} [1 - R_j(t)]^{l_i} R_j(t)^{1-r_i},
$$

$$
t \in (-\infty, \infty),
$$

where $\bar{m} = k_n - m$.

2. System of VTS Zatoka radars

Radar’s system is the basic subsystem of whole VTS system and also part of identification and watching system at the Gulf of Gdansk region. The purpose of that system is assuring real time information about ships’ traffic in that region [3]. VTS Zatoka system works involving five shores based radars, which are put in following places:

- Lighthouse Hel, (radar height 42,5 m a.s.l.);
- Port of Gdynia Harbourmaster Office building (HMO), (32,5 m a.s.l.);
- Northern Port of Gdansk Harbourmaster Office building, (66 m a.s.l.);
- Western Hills?, (17,5 m a.s.l.);
- Lighthouse Krynica Morska, (26 m a.s.l.).

Radars work permanently and their range cover whole responsibility area assigned to VTS Zatoka. Two, or even three, radars cover most part of Gulf of Gdansk simultaneously. That situation has great matter in case of failure of single radar.

For the VTS Zatoka systems’ radars, apart from standard equipment, additional Radar Data Processor (RDP) has been installed. RDP changes radar data from analogue to digital form. This digital information is next transferred to VTS centre by wireless line or light cable, which connect two Harbourmaster’s offices of Ports in Gdynia and Gdansk. Signals from radars, after preliminary treatment, are transferred to the VTS Centre. Then after final processing signals are sending out and visualized (with use of computer program ARAMIS) at VTS Centre itself, Harbourmaster’s offices of Gdynia and Gdansk ports and at Harbourmaster’s office of Krynica Morska port.

Scheme of the radar’s subsystem and data transmission is showed on Figure 5.

![Figure 5](image)

Figure 5. Scheme of VTS Zatoka radars’ system

3. VTS Zatoka radar system reliability

In order to analyse the considered system reliability we will firstly calculate reliability parameters of single radar.

![Figure 4](image)

Figure 4. Positions of VTS Zatoka shore based radars (dots)
3.1. Single radar reliability

VTS Zatoka system has been designed and constructed by Holland Institute of Traffic Technology (HITT). It came into service on the 1st of May 2003. Used radars were produced by Danish corporation Terma Electronic AS (frequency 9735 MHz). Mean time between failure and mean time to repair given by HITT have been used to construct VTS Zatoka radars’ subsystem reliability model (Table 1). Times to repair given in Table 1, concern the situation in which service team is near damaged radar. In fact, sustaining service team near each radar is very expensive. For further consideration we assume that service team is in Tricity i.e. in Gdynia, Sopot or Gdansk. Taking into account access time, time needed to fix what device is damaged and mean time of device’ exchange, mean time to repair of the single radar (MTTR₁) is about 3 and a half hours. After considering frequency of failures of particular parts of radars, the mean time of exchanging damaged part is about 40 minutes.

To find single radar reliability we assume that the radar is a series system. This means that the failure of one component of radar causes the failure of the whole radar. Two radars placed at Harbourmaster office buildings have five elements (antenna, single radar transmitter, receiver, video processor, radar processor), three others additionally have data transmitter. We assume that component reliability functions are exponential and given by the equation

\[ R_i(t) = \exp(-\lambda_i \cdot t), \ t \geq 0. \]  \hspace{1cm} (11)

When we put data from Table 1 to (11), and then to equation (1), as a result we obtain the single radar reliability function for radars at harbourmaster office buildings

\[ R_H(t) = \exp(-0.000229 \cdot t), \ t \geq 0, \]  \hspace{1cm} (12)

and for three others radars

\[ R_O(t) = \exp(-0.000235 \cdot t), \ t \geq 0. \]  \hspace{1cm} (13)

Mean time between failures of single radar is given by equation

\[ MTBF = \int_0^\infty R(t) \, dt. \]  \hspace{1cm} (14)

According to equations (12)-(14) and to Table 1, we obtain mean time between failures of single radar at harbourmaster office buildings

\[ MTBF_H = \frac{1}{0.000229} = 4366h \approx 182\text{days}, \]

and mean time between failures of three other radars

\[ MTBF_O = \frac{1}{0.000235} = 4255h \approx 177\text{days}. \]

3.2. Reliability of radar system

In order to evaluate radars system reliability, we can use different approaches. First, we can assume that subsystem is series. VTS Zatoka radars’ subsystem is working when all five radars are working. According to equation (3) with parameters

\[ n = 5, \ q_1 = \frac{2}{5}, \ q_2 = \frac{3}{5}, \]

and equations (12) and (13), we obtain the system reliability function

\[ R_S(t) = [\exp(-0.000229)]^5[\exp(-0.000235)]^3 \]

\[ = \exp(-0.001163), \ t \geq 0. \]  \hspace{1cm} (15)

Mean time between failures of that system is given by equation

\[ MTBF_S = \int_0^\infty R_S(t) \, dt = \frac{1}{0.001163} \]

\[ = 860h \approx 36\text{days}. \]  \hspace{1cm} (16)

System availability is given by equation [1]

\[ G = \frac{MTBF_S}{MTBF_S + MTTR_S} \]  \hspace{1cm} (17)

and after substituting \( MTBF_S = 860h, \ MTTR_S = 3.5h, \) amounts
Another way of describing reliability of radars' subsystem is assumption that system is “\( m \) out of \( n \)”. We can assume that system is working if at least four out of five radars are working. If we take into account that particular radars are series systems we obtain a non-homogenous series-\( "4 \) out of \( 5" \) system.

The above assumption is acceptable because ranges of any four radars covered fairways to ports in Gdansk and Gdynia and most traffic (nearly entire) is concentrated in those fairways. According to equations (10) and Table 1, reliability function of such system is given by equation

\[
R_{SMN}(t) = \bar{R}_{5,5,5,5,6,6,6}(t) = 2\exp(-0.000934) \\
+ 3\exp(-0.000928) - 4\exp(-0.001163) . \tag{18}
\]

\( R_{SMN}(t) \) function is showed on Figure 4. Mean time to failure of series-\( "m \) out of \( n" \) system \( MTBF_{SMN} \) is given by equation

\[
MTBF_{SMN} = \int_0^\infty R_{SMN}(t) \, dt . \tag{19}
\]

The mean time to failure of above described system according to equations (18) and (19) equals

\[
MTBF_{SMN} = \int_0^\infty \bar{R}_{5,5,5,5,6,6,6}(t) \, dt = 2 \cdot \frac{1}{0.000934} \\
+ 3 \cdot \frac{1}{0.000928} - 4 \cdot \frac{1}{0.001163} \]

\[= 1935h \approx 81 \text{ days}. \tag{20}\]

The availability of the series-\( "m \) out of \( n" \) system is given by equation

\[
G_{SMN} = \frac{MTBF_{SMN}}{MTBF_{SMN} + MTTR} \tag{21}
\]

and hence

\[
G_{SMN} = \frac{1935}{1935 + 3.5} = 0.9982
\]

As we can see system defined as series-\( "m \) out of \( n" \) has both higher reliability and availability than a series system.

4. Conclusion

As we can see from the performed analysis evaluation of the system reliability depends on taken assumptions. Reliability functions are significantly different one from the other (Figure 6), so choosing proper method of describing of system reliability structure is very important.

Whatever method was chosen, thanks to reliability of components of radars, VTS Zatoka radars system is highly reliable. Access to spare parts and organization of service has significant matter for availability of the subsystem. In order to sustain acceptable availability it is necessary to provide the service support located in Tri-City. It allows for quick reaction in case of failures and for repairing damaged parts of radars.

References

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