**Reliability and availability of a shipyard ship-rop e elevator in variable operation conditions**

**Keywords**
transportation system, reliability, risk, availability, system operation process

**Abstract**
In the paper the environment and infrastructure influence of the ship-rop e elevator operating in Naval Shipyard in Gdynia on its operation processes is considered. The results are presented on the basis of a general model of technical systems operation processes related to their environment and infrastructure. The elevator operation process is described and its statistical identification is given. Next, the elevator is considered in varying in time operation conditions with different its components’ reliability functions in different operation states. Finally, the reliability, risk and availability evaluation of the elevator in variable operation conditions is presented.

1. **Description of the ship-rop e elevator in Naval Shipyard in Gdynia**

Ship-rop e elevators are used to dock and undock ships coming to shipyards for repairs. The elevator utilized in the Naval Shipyard in Gdynia, with the scheme presented in Figure 4, is composed of a steel platform-carriage placed in its syncline (hutch). The platform is moved vertically with 10 rope-hoisting winches fed by separate electric motors. Motors are equipped in ropes “Bridon” with the diameter 47 mm each rope having a maximum load of 300 tonnes. During ship docking the platform, with the ship settled in special supporting carriages on the platform, is raised to the wharf level (upper position). During undocking, the operation is reversed. While the ship is moving into or out of the syncline and while stopped in the upper position the platform is held on hooks and the loads in the ropes are relieved. Since the platform-carriage and electric motors are highly reliable in comparison to the ropes, which work in extremely aggressive conditions, in our further analysis we will discuss the reliability of the rope system only.

![Figure 1. The ship-rop e transportation system (upper position)](image-url)
The system under consideration is composed of 10 ropes linked in series. Each of the ropes is composed of 22 strands: 10 outer and 12 inner.

![Figure 2. The scheme of the ship-rope elevator](image)

The assumption that ropes satisfy the technical conditions when at least one of its strands satisfies these conditions is not always true. In reality it is said that a rope is failed after some number of strands use. Therefore better, closer in reality approach to the system reliability evaluation is assumption that the ship-rope transportation system is “m out of n”-series system. Further we assume that \( m = 5 \).

2. Operation process and its statistical identification

Considering the tonnage of the docked and undocked ships by the rope elevator in Naval Shipyard in Gdynia we can divide the system’s load, similarly as in the previous ships’ transportation system, into six groups and due to fact that the rope elevator system depends mainly on the tonnage of docking ships we can distinguish the following (\( v = 6 \)) operation states of the rope elevator system operation process:

- an operation state \( z_1 \) – without loading (the system is not working),
- an operation state \( z_2 \) – loading over 0 up to 500 tonnes,
- an operation state \( z_3 \) – loading over 500 up to 1000 tonnes,
- an operation state \( z_4 \) – loading over 1000 up to 1500 tonnes,
- an operation state \( z_5 \) – loading over 1500 up to 2000 tonnes,
- an operation state \( z_6 \) – loading over 2000 up to 2500 tonnes.

In all six operational states system has the same structure. There are 10 rope-hoisting winches equipped in identical ropes and each of the ropes is composed of 22 strands. We assume that the rope is “5 out of 22” system, so we consider the ship-rope elevator as a regular “5 out of 22”-series system.

![Figure 3. The scheme of the rope-hoisting winches placing](image)

On the basis of the statistical data coming from experts using the shipyard ship-rope elevator in Naval Shipyard in Gdynia [6] the transition probabilities \( p_{bi} \) from the operation state \( z_b \) into the operation state \( z_l, \ b, l = 1, ..., 6, \ b \neq l, \) were evaluated. Their approximate evaluations are given in the matrix below.

\[
\begin{bmatrix}
0 & 0.2931 & 0.2931 & 0.2414 & 0.1207 & 0.0517 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

On the basis of the realizations of the operation process \( Z(t) \) conditional sojourn times \( \theta_{bi} \), \( b, l = 1, 2, ..., 6, \ b \neq l, \) in the state \( z_b \) while the next transition is to the state \( z_l \), given in [6], there were formulated hypotheses about the distributions of the conditional sojourn times \( \theta_{bi} \). These hypotheses allows us to estimate the conditional mean values \( M_{bi} = E[\theta_{bi}], \ b, l = 1, 2, ..., 6, \ b \neq l, \) of the lifetimes in the particular operation states:

\[
M_{12} = 3057.06, \ M_{13} = 3319.12, \ M_{14} = 10406.07, \\
M_{15} = 4687.86, \ M_{16} = 5540.00, \\
M_{21} = 58.00, \ M_{31} = 37.18, \ M_{41} = 183.21,
\]
\[ M_{51} = 124.50, \quad M_{61} = 270.00. \]

Hence, by [10], the unconditional mean sojourn times in the particular operation states are determined from the formula

\[ M_b = E[\theta_b] = \sum_{i=1}^{k_b} p_{i} M_{bi}, \quad b = 1, \ldots, 6, \]

and takes values:

\[ M_1 = 5233.13, \quad M_2 = 58.00, \quad M_3 = 37.18, \]
\[ M_4 = 183.21, \quad M_5 = 124.50, \quad M_6 = 270.00. \]

Since from the system of equations below [10], [11]

\[
\begin{align*}
[\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] &= [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] [p_{bi}] \\
\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 &= 1,
\end{align*}
\]

we get

\[ \pi_1 = 0.5, \quad \pi_2 = 0.14655, \quad \pi_3 = 0.14655 \]
\[ \pi_4 = 0.1207, \quad \pi_5 = 0.06035, \quad \pi_6 = 0.02585. \]

Then the limit values of the transient probabilities \( p_b(t) \) at the operational states \( z_b \), according to results given in [5],[7], are equal to:

\[ p_1 = 0.9810, \quad p_2 = 0.0032, \quad p_3 = 0.0021, \]
\[ p_4 = 0.0083, \quad p_5 = 0.0028, \quad p_6 = 0.0026. \tag{1} \]

3. Reliability of the shipyard ship-rope elevator

According to rope reliability data given in their technical certificates and experts’ opinions based on the nature of strand failures the following reliability states have been distinguished:

- a reliability state 3 – a strand is new, without any defects,
- a reliability state 2 – the number of broken wires in the strand is greater than 0% and less than 25% of all its wires, or corrosion of wires is greater than 0% and less than 25%,
- a reliability state 1 – the number of broken wires in the strand is greater than or equal to 25% and less than 50% of all its wires, or corrosion of wires is greater than or equal to 25% and less than 50%,
- a reliability state 0 – otherwise (a strand is failed).

We consider the strands as basic components of the system. The system of ropes is in the reliability state subset \{1,2,3\}, \{2,3\}, \{3\}, when all of its ropes are in this state subset and each of the ropes is in the reliability state subset \{1,2,3\}, \{2,3\}, \{3\}, if at least 5 of 22 strands are in this state subset. Thus, we conclude that the ship-rope elevator is a regular 4-series-linked subsystems (ropes) with \( l_n = 10 \) series-linked subsystems (ropes) with \( l_n = 22 \) parallel-linked components (strands).

Then, taking into account above remarks, we obtain the reliability function of the considered ship-rope elevator given by the vector

\[ R(t, \cdot) = [1, R(t, 1), R(t, 2), R(t, 3)] \]

\[ = [1, \bar{R}_{10, 22}^{(5)} (t, 1), \bar{R}_{10, 22}^{(5)} (t, 2), \bar{R}_{10, 22}^{(5)} (t, 3)], \quad t \in 0, \infty. \tag{2} \]

We assume strands as a basic components of a system with the reliability functions given by the vector

\[ R(t, \cdot) = [R(t, 0), R(t, 1), R(t, 2), R(t, 3)], \quad t \in 0, \infty, \]

with the co-ordinates

\[ R(t, u) = P(S(t) \geq u | S(0) = 3) = P(T(u) > t) \]

for \( t < 0, \infty \), \( u = 0,1,2,3 \), and \( R(t, 0) = 1 \). \( T(u) \) is independent random variable representing the lifetime of system components in the reliability state subset \{0,1,2,3\}, while they were at the reliability state 3 at the moment \( t = 0 \) and \( S(t) \) are components’ reliability states at the moment \( t \), \( t < 0, \infty \).

Moreover we assume that the components of the ship-rope elevator i.e. strands have multi-state reliability functions

\[ R^{(b)}(t, \cdot) = [1, R^{(b)}(t, 1), R^{(b)}(t, 2), R^{(b)}(t, 3)], \]

with exponential co-ordinates \( R^{(b)}(t, 1), R^{(b)}(t, 2) \) and \( R^{(b)}(t, 3) \) different in various operation states \( z_b, \quad b = 1,2,\ldots,6. \)

At the system operational state \( z_1 \) the strands in the ropes have following conditional reliability functions co-ordinates:

\[ R^{(1)}(t, 1) = \exp[-0.1613t], \]
\[ R^{(1)}(t, 2) = \exp[-0.2041t], \]

\[ R^{(1)}(t, 3) = \exp[-0.1613t]. \]
Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state $z_1$ is given by:

$$
[R(t,\cdot)]^{(1)} = [1, [R(t,1)]^{(1)}, [R(t,2)]^{(1)}, [R(t,3)]^{(1)}],
$$

where

$$
[R(t,1)]^{(1)} = [R^{(5)}(t,1)]^{(1)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 3 \\ i \end{array} \right) \exp[-0.1613it]]
$$

$$
[1 - \exp[-0.1613it]]^{22-i} ]^{10},
$$

$$
[R(t,2)]^{(1)} = [R^{(5)}(t,2)]^{(1)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 3 \\ i \end{array} \right) \exp[-0.2041it]]
$$

$$
[1 - \exp[-0.2041it]]^{22-i} ]^{10},
$$

$$
[R(t,3)]^{(1)} = [R^{(5)}(t,3)]^{(1)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 3 \\ i \end{array} \right) \exp[-0.2326it]]
$$

$$
[1 - \exp[-0.2326it]]^{22-i} ]^{10} \text{ for } t \geq 0.
$$

For $z_2$, the strands in the ropes have the following conditional reliability functions co-ordinates:

$$
[R(t,\cdot)]^{(2)} = [1, [R(t,1)]^{(2)}, [R(t,2)]^{(2)}, [R(t,3)]^{(2)}],
$$

where

$$
[R(t,1)]^{(2)} = [R^{(5)}(t,1)]^{(2)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 2 \\ i \end{array} \right) \exp[-0.2041it]]
$$

$$
[1 - \exp[-0.2041it]]^{22-i} ]^{10},
$$

$$
[R(t,2)]^{(2)} = [R^{(5)}(t,2)]^{(2)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 2 \\ i \end{array} \right) \exp[-0.2564it]]
$$

$$
[1 - \exp[-0.2564it]]^{22-i} ]^{10},
$$

$$
[R(t,3)]^{(2)} = [R^{(5)}(t,3)]^{(2)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 2 \\ i \end{array} \right) \exp[-0.2941it]]
$$

$$
[1 - \exp[-0.2941it]]^{22-i} ]^{10} \text{ for } t \geq 0.
$$

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (3)-(5), according to [4], [8], at the operation state $z_1$, in years, are respectively given by:

$$
\mu_1(1) \equiv 6.4415, \mu_1(2) \equiv 5.0907, \mu_1(3) \equiv 4.4669,
$$

$$
\sigma_1(1) \equiv 1.0563, \sigma_1(2) \equiv 0.8345, \sigma_1(3) \equiv 0.7323,
$$

and further, using (6), from [8] it follows that the conditional lifetimes in the particular reliability states at the operation state $z_2$, in years, are:

$$
\bar{\mu}_1(1) \equiv 1.3508, \bar{\mu}_1(2) \equiv 0.6239, \bar{\mu}_1(3) \equiv 4.4669.
$$

At the system operational state $z_2$ the strands in the ropes have following conditional reliability functions co-ordinates:

$$
[R(t,\cdot)]^{(2)} = [1, [R(t,1)]^{(2)}, [R(t,2)]^{(2)}, [R(t,3)]^{(2)}],
$$

where

$$
[R(t,1)]^{(2)} = [R^{(5)}(t,1)]^{(2)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 2 \\ i \end{array} \right) \exp[-0.2041it]]
$$

$$
[1 - \exp[-0.2041it]]^{22-i} ]^{10},
$$

$$
[R(t,2)]^{(2)} = [R^{(5)}(t,2)]^{(2)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 2 \\ i \end{array} \right) \exp[-0.2564it]]
$$

$$
[1 - \exp[-0.2564it]]^{22-i} ]^{10},
$$

$$
[R(t,3)]^{(2)} = [R^{(5)}(t,3)]^{(2)} = [1 - \sum_{i=1}^{4} \left( \begin{array}{c} 2 \\ i \end{array} \right) \exp[-0.2941it]]
$$

$$
[1 - \exp[-0.2941it]]^{22-i} ]^{10} \text{ for } t \geq 0.
$$

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (8)-(10), according to [8] at the operation state $z_2$ are respectively given by:

$$
\mu_2(1) \equiv 5.0907, \mu_2(2) \equiv 4.0523,
$$

$$
\mu_2(3) \equiv 3.5335,
$$

$$
\sigma_2(1) \equiv 0.8345, \sigma_2(2) \equiv 0.6639,
$$

$$
\sigma_2(3) \equiv 0.5000.
\[ \sigma_2(3) \cong 0.5744, \]  
and further, using (11), from [8] it follows that the conditional lifetimes in the particular reliability states at the operation state \( z_2 \) are:

\[ \bar{\mu}_2(1) \cong 1.0384, \bar{\mu}_2(2) \cong 0.5188, \bar{\mu}_2(3) \cong 3.5335. \]

At the system operational state \( z_3 \) the strands in the ropes have following conditional reliability functions co-ordinates:

\[ R^{(3)}(t,1) = \exp[-0.2222t], \]
\[ R^{(3)}(t,2) = \exp[-0.2857t], \]
\[ R^{(3)}(t,3) = \exp[-0.3226t] \quad \text{for} \quad t \geq 0. \]

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state \( z_3 \) is given by:

\[ [\bar{R}(t,\cdot)]^{(3)} = [1, [\bar{R}(t,1)]^{(3)}, [\bar{R}(t,2)]^{(3)}, [\bar{R}(t,3)]^{(3)}], \]
where

\[ [\bar{R}(t,1)]^{(3)} = [\bar{R}_{10,22}^{(5)}(t,1)]^{(3)} \]
\[ = [1 - \sum_{i=1}^{4} \binom{2}{i} \exp[-0.2222i t] \cdot [1 - \exp[-0.2222t]]^{22-i}]^{(3)}, \]  
\[ [\bar{R}(t,2)]^{(3)} = [\bar{R}_{10,22}^{(5)}(t,2)]^{(3)} \]
\[ = [1 - \sum_{i=1}^{4} \binom{2}{i} \exp[-0.2857i t] \cdot [1 - \exp[0.2857t]]^{22-i}]^{(3)}, \]  
\[ [\bar{R}(t,3)]^{(3)} = [\bar{R}_{10,22}^{(5)}(t,3)]^{(3)} \]
\[ = [1 - \sum_{i=1}^{4} \binom{2}{i} \exp[-0.3226i t] \cdot [1 - \exp[-0.3226t]]^{22-i}]^{(3)}, \] for \( t \geq 0. \]

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (13)-(15), according to results given in [8], at the operation state \( z_3 \), in years are equal to:

\[ \mu_3(1) \cong 4.6760, \mu_3(2) \cong 3.6367, \]
\[ \mu_3(3) \cong 3.2207, \]  
\[ \sigma_3(1) \cong 0.7665, \sigma_3(2) \cong 0.5956, \]
\[ \sigma_3(3) \cong 0.5273, \]

and further, from (16) and [8] it follows that the conditional lifetimes in the particular reliability states at the operation state \( z_3 \) are:

\[ \bar{\mu}_3(1) \cong 1.0393, \bar{\mu}_3(2) \cong 0.4160, \bar{\mu}_3(3) \cong 3.2207. \]

At the system operational state \( z_4 \) the strands in the ropes have following conditional reliability functions co-ordinates:

\[ R^{(4)}(t,1) = \exp[-0.2702t], \]
\[ R^{(4)}(t,2) = \exp[-0.3508t], \]
\[ R^{(4)}(t,3) = \exp[-0.4167t] \quad \text{for} \quad t \geq 0. \]

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state \( z_4 \) is given by:

\[ [\bar{R}(t,\cdot)]^{(4)} = [1, [\bar{R}(t,1)]^{(4)}, [\bar{R}(t,2)]^{(4)}, [\bar{R}(t,3)]^{(4)}], \]
where

\[ [\bar{R}(t,1)]^{(4)} = [\bar{R}_{10,22}^{(5)}(t,1)]^{(4)} \]
\[ = [1 - \sum_{i=1}^{4} \binom{2}{i} \exp[-0.2702i t] \cdot [1 - \exp[-0.2702t]]^{22-i}]^{(4)}, \]  
\[ [\bar{R}(t,2)]^{(4)} = [\bar{R}_{10,22}^{(5)}(t,2)]^{(4)} \]
\[ = [1 - \sum_{i=1}^{4} \binom{2}{i} \exp[-0.3508i t] \cdot [1 - \exp[-0.3508t]]^{22-i}]^{(4)}, \]  
\[ [\bar{R}(t,3)]^{(4)} = [\bar{R}_{10,22}^{(5)}(t,3)]^{(4)} \]
\[ = [1 - \sum_{i=1}^{4} \binom{2}{i} \exp[-0.5188i t] \cdot [1 - \exp[-0.5188t]]^{22-i}]^{(4)}, \] for \( t \geq 0. \]
\[ [1 - \sum_{i=1}^{4} \left( \frac{1}{i!} \right) \exp[-0.4167it] ] \]

\[ [1 - \exp[-0.4167ir]]^{22-i} \] for \( t \geq 0 \).

The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (18)-(20), according to results in [8], at the operation state \( z_4 \) are respectively given by:

\[ \mu_4(1) \equiv 3.8453, \mu_4(2) \equiv 2.9618, \]
\[ \mu_4(3) \equiv 2.4934, \] (21)

\[ \sigma_4(1) \equiv 0.6301, \sigma_4(2) \equiv 0.4846, \]
\[ \sigma_4(3) \equiv 0.4074, \] (22)

and further, using (21), from [8] it follows that the conditional lifetimes in the particular reliability states at the operation state \( z_4 \) are:

\[ \mu_4(1) \equiv 3.1173, \mu_4(2) \equiv 2.1819, \]
\[ \mu_4(3) \equiv 1.7664, \] (26)

\[ \sigma_4(1) \equiv 0.5103, \sigma_4(2) \equiv 0.3574, \]
\[ \sigma_4(3) \equiv 0.2894, \] (27)

At the system operational state \( z_5 \) the strands in the ropes have following conditional reliability functions co-ordinates:

\[ R^{(5)}(t,1) = \exp[-0.3333r], \]
\[ R^{(5)}(t,2) = \exp[-0.4762r], \]
\[ R^{(5)}(t,3) = \exp[-0.5882r] \] for \( t \geq 0 \).

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state \( z_5 \) is given by:

\[ [\overline{R}(t,\cdot)]^{(5)} = [1, [\overline{R}(t,1)]^{(5)}, [\overline{R}(t,2)]^{(5)}, [\overline{R}(t,3)]^{(5)} \], \]

where

\[ [\overline{R}(t,1)]^{(5)} = [\overline{R}^{(5)}_{10,22}(t,1)]^{(5)} \]
\[ = [1 - \sum_{i=1}^{4} \left( \frac{1}{i!} \right) \exp[-0.3333it] ] \]
\[ [1 - \exp[-0.3333ir]]^{22-i} \] for \( t \geq 0 \).

\[ [\overline{R}(t,2)]^{(5)} = [\overline{R}^{(5)}_{10,22}(t,2)]^{(5)} \]

\[ [\overline{R}(t,3)]^{(5)} = [\overline{R}^{(5)}_{10,22}(t,3)]^{(5)} \]

\[ \mu_5(1) \equiv 3.1173, \mu_5(2) \equiv 2.1819, \]
\[ \mu_5(3) \equiv 1.7664, \] (26)

\[ \sigma_5(1) \equiv 0.5103, \sigma_5(2) \equiv 0.3574, \]
\[ \sigma_5(3) \equiv 0.2894, \] (27)

and further, using (26), from [8] it follows that the conditional lifetimes in the particular reliability states at the operation state \( z_5 \) are:

\[ \mu_5(1) \equiv 0.9354, \mu_5(2) \equiv 0.4155, \]
\[ \mu_5(3) \equiv 1.7664. \] (28)

At the system operational state \( z_6 \) the strands in the ropes have following conditional reliability functions co-ordinates:

\[ R^{(6)}(t,1) = \exp[-0.4348r], \]
\[ R^{(6)}(t,2) = \exp[-0.7143r], \]
\[ R^{(6)}(t,3) = \exp[-0.9091r] \] for \( t \geq 0 \).

Thus the conditional multi-state reliability function of the ship-rope elevator at the operational state \( z_6 \) is given by:

\[ [\overline{R}(t,\cdot)]^{(6)} = [1, [\overline{R}(t,1)]^{(6)}, [\overline{R}(t,2)]^{(6)}, [\overline{R}(t,3)]^{(6)} \], \]

where

\[ [\overline{R}(t,1)]^{(6)} = [\overline{R}^{(6)}_{10,22}(t,1)]^{(6)} \]
The expected values and standard deviations of the ship-rope elevator conditional lifetimes in the reliability state subsets calculated from the above result given by (28)-(30), and from [8] at the operation state $z_6$ are respectively given in years by:

$$
\mu_6(1) \cong 2.3896, \quad \mu_6(2) \cong 1.4546, \quad \mu_6(3) \cong 1.1429,
$$

and further, from (31) and [8] it follows that the conditional lifetimes in the particular reliability states at the operation state $z_6$ in years are equal to:

$$
\bar{\mu}_6(1) \cong 0.9350, \quad \bar{\mu}_6(2) \cong 0.3117, \quad \bar{\mu}_6(3) \cong 1.1429.
$$

In the case when the operation time is large enough its unconditional multi-state reliability function of the ground ship-rope transporter is given by the vector

$$
\bar{\mathbf{R}}(t, \cdot) = [1, \bar{\mathbf{R}}(t,1), \bar{\mathbf{R}}(t,2), \bar{\mathbf{R}}(t,3)], \quad t \in (0, \infty),
$$

where according to [5], [11], the vector co-ordinates are given respectively by:

$$
\bar{\mathbf{R}}(t,u) = \sum_{i=1}^{6} p_i [\bar{\mathbf{R}}(t,u)]^{(i)} \quad \text{for} \quad t \geq 0, \quad u = 1, 2, 3,
$$

where $[\bar{\mathbf{R}}(t,u)]^{(i)}$, $i = 1, \ldots, 6$, are given by (3)-(5), (8)-(10), (13)-(15), (18)-(20), (23)-(25), (28)-(30).

The mean values and the standard deviations of the ground ship-rope transporter unconditional lifetimes in the reliability state subsets, according to [10], [11] and after considering (6)-(7), (11)-(12), (16)-(17), (21)-(22), (26)-(27), (31)-(32) and (1), respectively are:

$$
\mu(1) = \sum_{i=1}^{6} \mu_i(1) \equiv 6.3887, \quad \sigma(1) \equiv 1.1336,
$$

$$
\mu(2) = \sum_{i=1}^{6} \mu_i(2) \equiv 5.0463, \quad \sigma(2) \equiv 0.9041,
$$

$$
\mu(3) = \sum_{i=1}^{6} \mu_i(3) \equiv 4.4266, \quad \sigma(3) \equiv 0.7964.
$$

Next, the unconditional mean values of the ground ship-rope transporter lifetimes in the particular reliability states, by [8] and considering (34)-(36), in years are:

$$
\bar{\mu}(1) = \bar{\mu}_6(1) - \mu(1) = 3.4244, \quad \bar{\mu}(2) = \bar{\mu}_6(2) - \mu(2) = 4.9433, \quad \bar{\mu}(3) = \bar{\mu}_6(3) - \mu(3) = 3.2584.
$$

If the critical reliability state is $r = 2$, then according to [4], the system risk function takes the form

$$
\tau = r^{-1}(\delta) \equiv 3.577 \text{ years} \equiv 3 \text{ years} 205 \text{ days}.
$$
4. Availability of the shipyard ship-rope elevator

In this point the asymptotic evaluation of the basic reliability and availability characteristics of renewal systems with non-ignored time of renovation are determined in an example of the shipyard ship-rope elevator. The theoretical results of multi-state systems availability analysis can be found in [1], [4].

Assuming that the ship-rope elevator is repaired after its failure and that the time of the system renovation is not ignored and it has the mean value \( \mu_0(2) = 0.0014 \leq 12 \) hours and the standard deviation \( \sigma_0(2) = 0.0002 \leq 2 \) hours, applying results from [4], we obtain the following results:

i) the distribution function of the time \( \bar{S}_N(2) \) until the \( N \)th system’s renovation, for sufficiently large \( N \), has approximately normal distribution \( N(5.04777N,0.9041\sqrt{N}) \), i.e.,

\[
\bar{F}^{(N)}(t,2) = P(\bar{S}_N(2) < t) = F_{N(0,1)}\left(\frac{t - 5.04777N}{0.9041\sqrt{N}}\right),
\]

\( t \in (-\infty, \infty), \quad N = 1,2,... \),

ii) the expected value and the variance of the time \( \bar{S}_N(2) \) until the \( N \)th system’s renovation take respectively forms

\[
E[\bar{S}_N(2)] = 5.04777N, \quad D[\bar{S}_N(2)] = 0.8174N,
\]

iii) the distribution function of the time \( \bar{S}_N(2) \) until the \( N \)th exceeding the reliability critical state 2 of this system takes form

\[
\bar{F}^{(N)}(t,2) =
\]

\[
P(\bar{S}_N(2) < t) = F_{N(0,1)}\left(\frac{t - 5.04777N + 0.0014}{0.9041\sqrt{N}}\right),
\]

\( t \in (-\infty, \infty), \quad N = 1,2,... \),

iv) the expected value and the variance of the time \( \bar{S}_N(2) \) until the \( N \)th exceeding the reliability critical state 2 of this system take respectively forms

\[
E[\bar{S}_N(2)] = 5.0463N + 0.0014(N - 1),
\]

\[
D[\bar{S}_N(2)] = 0.8174N,
\]

v) the distribution of the number \( \bar{N}(t,2) \) of system’s renovations up to the moment \( t, t \geq 0 \), is of the form

\[
P(\bar{N}(t,2) = N) \equiv F_{N(0,1)}\left(\frac{5.04777N - t}{0.4024\sqrt{t}}\right)
\]

\[
- F_{N(0,1)}\left(\frac{5.04777(N + 1) - t}{0.4024\sqrt{t}}\right), \quad N = 1,2,... ,
\]

vi) the expected value and the variance of the number \( \bar{N}(t,2) \) of system’s renovations up to the moment \( t, t \geq 0 \), take respectively forms

\[
\bar{H}(t,2) \equiv 0.1981t, \quad \bar{D}(t,2) \equiv 0.0064t,
\]

vii) the distribution of the number \( \bar{N}(t,2) \) of exceeding the reliability critical state 2 of this system up to the moment \( t, t \geq 0 \), is of the form

\[
P(\bar{N}(t,2) = N) \equiv F_{N(0,1)}\left(\frac{5.04777N - t - 0.0014}{0.4024\sqrt{t} + 0.0014}\right)
\]

\[
- F_{N(0,1)}\left(\frac{5.04777(N + 1) - t - 0.0014}{0.4024\sqrt{t} + 0.0014}\right), \quad N = 1,2,... ,
\]

viii) the expected value and the variance of the number \( \bar{N}(t,2) \) of exceeding the reliability critical state 2 of this system up to the moment \( t, t \geq 0 \), are respectively given by

\[
\bar{H}(t,2) \equiv 0.1981t + 0.0003,
\]

\[
\bar{D}(t,2) \equiv 0.0064(t + 0.0014),
\]

ix) the availability coefficient of the system at the moment \( t \) is given by the formula

\[
\bar{H}(t) \equiv 0.1981t + 0.0003.
\]
\[ K(t, \tau, 2) \equiv 0.9997, \ t \geq 0, \]  
\[ x) \text{ the availability coefficient of the system in the time interval } < t, t + \tau), \tau > 0, \text{ is given by the formula} \]
\[ K(t, \tau, 2) \equiv 0.1981 \int_{0}^{\infty} \bar{R}(t, \tau) dt, \ t \geq 0, \tau > 0, \]
\[ \text{where the reliability function of a system at the critical state } \bar{R}(t, \tau) \text{ is given by the formula (33).} \]

5. Conclusion

In the paper an analytical model of port transportation systems environment and infrastructure influence on their operation process is presented. The theoretical results of reliability, risk and availability evaluation of industrial systems in variable operation conditions are applied to the shipyard ship-rope elevator in Naval Shipyard in Gdynia. These results may be considered as an illustration of the proposed methods possibilities of application in rope transportation systems reliability analysis. Other technical systems reliability evaluation related to their operation process are presented for example in [3], [11]. The obtained evaluations may be discussed as an example in transportation systems reliability characteristics evaluation, especially during the design and while planning and improving its operation process safety and effectiveness.

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References

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