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**Statistical identification and prediction of the port oil pipeline system’s operation process and its reliability and risk evaluation**

**Keywords**
semi-markov processes, system operations process, reliability

**Abstract**
In the paper a Semi-markov processes are used to construct a general model of complex industrial systems’ operation processes. Main characteristics of this model are determined as well. In particular case, for a port oil pipeline transportation system, its operation states are defined, the relationships between them are fixed and particular model of its operation process is constructed and its main characteristics are determined. Further, the joint model of the system operation process and the system reliability is defined and applied to the reliability and risk evaluation of the port oil pipeline transportation system.

**1. Introduction**
The complexity of the systems’ operation processes and their influence on changing in time the systems’ structures and their components’ reliability characteristics is often very difficult to fix and to analyse. A convenient tool for solving this problem is semi-markov modelling of the systems operation processes proposed in the paper. Therefore, the common usage of the system’s reliability evaluation methods and semi-markov modelling the system’s exploitation process in order to construct a general system reliability model related to its operation process is proposed in the paper. Moreover, statistical methods of the general model unknown parameters estimation are proposed and applied in pipeline transport.

**3. Modelling of system operation process**
We assume that the system during its operation process has \( v \) different operation states. Thus we can define the system operation process \( Z(t), t \in <0,+) \), as the process with discrete operation states from the set

\[
Z = \{ z_1, z_2, \ldots, z_v \}.
\]

In practice a convenient assumption is that \( Z(t) \) is a semi-Markov process [1] with its conditional sojourn times \( \theta_{bl} \) at the operation state \( z_b \) when its next operation state is \( z_l, b, l = 1,2,\ldots,v, b \neq l \). In this case the process \( Z(t) \) may be described by:
- the vector of probabilities of the system operation process initial states
  \[
  [p_b(0)]_{1xv} = [p_1(0), p_2(0), \ldots, p_v(0)],
  \]
  where
  \[
  p_b(0) = P(Z(0) = z_b) \quad \text{for} \quad b = 1,2,\ldots,v,
  \]
- the matrix of probabilities of the system operation process transitions between the operation states
  \[
  [p_{bl}]_{v \times v} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \vdots & \vdots & \ddots & \vdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix},
  \]
  where \( p_{bb} = 0 \) for \( b = 1,2,\ldots,v \),
- the matrix of the system operation process conditional sojourn times \( \theta_{bl} \) distribution functions
Kołowrocki Krzysztof, Soszyńska Joanna

Statistical identification and prediction of the port oil pipeline system’s operation process and its reliability evaluation

\[
[H_{bl}(t)]_{xv} = \begin{bmatrix}
H_{11}(t) & H_{12}(t) & \ldots & H_{1v}(t) \\
H_{21}(t) & H_{22}(t) & \ldots & H_{2v}(t) \\
\vdots & \vdots & \ddots & \vdots \\
H_{v1}(t) & H_{v2}(t) & \ldots & H_{vv}(t)
\end{bmatrix},
\] (2)

where

\[H_{bl}(t) = P(\theta_{bl} < t) \text{ for } b, l = 1,2,\ldots, v, \ b \neq l,\]

and

\[H_{bb}(t) = 0 \text{ for } b = 1,2,\ldots, v.\]

Under these assumptions, the mean values of the system operation process conditional sojourn times \(\theta_{bl}\) are given by

\[M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t), \ b, l = 1,2,\ldots, v, \ b \neq l.\] (3)

By the formula for total probability the unconditional distribution functions of the sojourn times \(\theta_{b}\) of the system operation process \(Z(t)\) at the operation states \(z_{b}\), \(b = 1,2,\ldots, v\), are given by

\[H_b(t) = \sum_{l=1}^v p_{bl} H_{bl}(t), \ b = 1,2,\ldots, v.\] (4)

Hence, the mean values \(E[\theta_b]\) of the system operation process unconditional sojourn times \(\theta_{b}\) in the particular operation states are given by

\[M_b = E[\theta_b] = \sum_{l=1}^v p_{bl} M_{bl}, \ b = 1,2,\ldots, v,\] (5)

where \(M_{bl}\) are defined by (3).

Moreover, it is well known [1] that the limit values of the system operation process transient probabilities at the particular operation states

\[p_b(t) = P(Z(t) = z_b) \text{ for } t < 0, +\infty), \ b = 1,2,\ldots, v,\]

are given by

\[p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{i=1}^v \pi_i M_i}, \ b = 1,2,\ldots, v,\] (6)

where \(M_b, b = 1,2,\ldots, v\), are defined by (5), whereas the probabilities \(\pi_b\) of the vector \([\pi_b]_{1v}\) satisfy the system of equations

\[\begin{bmatrix} \pi_b \\ \vdots \\ \pi_b \end{bmatrix} = \begin{bmatrix} [\pi_b]_{1v} \\ \vdots \\ [\pi_b]_{1v} \end{bmatrix}
\]

\[\sum_{i=1}^v \pi_i = 1.\] (7)

Other interesting characteristics of the operation process \(Z(t)\) possible to obtain are its total sojourn times \(\hat{\theta}_b\) in the particular operation states \(z_b\), \(b = 1,2,\ldots, v\). It is well known [5] that the system operation process total sojourn times \(\hat{\theta}_b\) in the particular operation states \(z_b\), for sufficiently large operation time \(\theta\), have approximately normal distribution with the expected value given by

\[E[\hat{\theta}_b] = p_b \theta, \ b = 1,2,\ldots, v,\] (8)

where \(p_b\) are given by (6).

3. The oil terminal in Dębogórze description

The oil terminal in Dębogórze [9] is designated for the reception from ships, the storage and sending by carriages or cars the oil products. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products such like petrol and oil [9].

The considered system is composed of three terminal parts \(A, B\) and \(C\), linked by the piping transportation systems. The scheme of this system is presented in Figure 1.

![Figure 1. The scheme of port oil transport system.](image_url)
Subsystem $S_2$ is built of two piping lines composed of steel pipe segments of the diameter 600 mm. The terminal part $B$ is connected with the terminal part $C$ by the subsystem $S_1$. The subsystem $S_1$ is built of one piping line composed of steel pipe segments of the diameter 500 mm and two piping lines composed of steel pipe segments of diameter 350 mm. The terminal part $C$ is designated for the loading the rail cisterns with oil products and for the wagon sending to the railway station of the Port of Gdynia and further to the interior of the country.

The Port Oil Transportation system consists of three subsystems $S_1$, $S_2$, $S_3$. (see Figure 1)

Subsystem $S_1$ consist of $k_n = 2$ two identical pipelines, each composed of $l_n = 178$ elements. In each pipeline there are:
- 176 pipe segments,
- 2 valves.

Subsystem $S_2$ consist of $k_n = 2$ two identical pipelines, each composed of $l_n = 719$ elements. In each pipeline there are:
- 717 pipe segments,
- 2 valves.

Subsystem $S_3$ consist of two pipelines of the first type and one second type, each composed of $l_n = 362$ elements. In each pipeline of the first type there are:
- 360 pipe segments (Ø=350mm),
- 2 valves.

In pipeline of the second type there are:
- 360 pipe segments (Ø=500mm),
- 2 valves.

4. The port oil pipeline transportation system operation process and its preliminary statistical identification

Taking into account the varying in time operation process of the considered system we distinguish the following as its eight operation states:

- an operation state $z_1$ — transport of one kind of medium from the terminal part $B$ to part $C$ using two out of three pipelines in subsystem $S_1$,

- an operation state $z_2$ — transport of one kind of medium from the terminal part $C$ (from carriages) to part $B$ using one out of three pipelines in subsystem $S_3$,

- an operation state $z_3$ — transport of one kind of medium from the terminal part $B$ through part $A$ to the pier using one out of two pipelines in subsystem $S_2$ and one out of two pipelines in subsystem $S_1$,

- an operation state $z_4$ — transport of two kinds of medium from the pier through parts $A$ and $B$ to part $C$ using one out of two pipelines in subsystem $S_1$, one out of two pipelines in subsystem $S_2$ and two out of three pipelines in subsystem $S_3$,
an operation state $z_6$ — transport of one kind of medium from the terminal part B to C using two out of three pipelines in subsystem $S_3$, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines in parts $S_1$ and one out of two pipelines in subsystem $S_2$.

• an operation state $z_7$ — lack of medium transport (system is not working)

• an operation state $z_8$ — transport of one kind of medium from the terminal part B to C using one out of three pipelines in part $S_3$, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines in part $S_3$.

In the Table 1 there is given an example realization of the conditional sojourn times in particular operation states of the oil pipeline system operation process.

**Table 1.** Realization of conditional sojourn times in operations states during one week.

<table>
<thead>
<tr>
<th>Condition al sojourn times $\theta_{bi}$</th>
<th>Realization of conditional sojourn times $\theta_{ai}$ in operation state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{17}$</td>
<td>8h 7h 15h 12h</td>
</tr>
<tr>
<td>$\theta_{71}$</td>
<td>1h 30m 15h 8h</td>
</tr>
<tr>
<td>$\theta_{57}$</td>
<td>7h 30m 21h 10m</td>
</tr>
<tr>
<td>$\theta_{75}$</td>
<td>0h 40m 41h</td>
</tr>
<tr>
<td>$\theta_{56}$</td>
<td>15h 10m</td>
</tr>
<tr>
<td>$\theta_{65}$</td>
<td>16h</td>
</tr>
</tbody>
</table>

It is assumed that one week of working of the system is a single realization of its operation process. The conditional sojourn times $\theta_{bi}$ at the operation states $z_b$ when its next operation state is $z_i$, $b, l = 1, 2, 3, 4, 5, 6, 7, 8$, $b \neq l$, of each single realization of the pipeline system operation process are given in separate line. In [4] there are collected realizations of the conditional sojourn times in particular operation states of considered system on the basis of a sample composed of $n = 10$ realizations.

To identify all parameters of the pipeline system operation process the statistical data about this process is needed. The statistical data that has been collected up to now is given in [4].

From data given in [4] the following basic operation process statistical data are fixed:

- the number of the pipeline system operation process states
\[ \nu = 8; \]

- the pipeline system operation process observation/experiment time

\[ \Theta = 70 \text{ days} = 10 \text{ weeks}; \]

- the number of the pipeline system operation process realizations

\[ n(0) = 10; \]

- the realization \( n_b(0) \) of the number of the pipeline system operation process transitions in the particular operation states \( z_b \) at the initial moment \( t = 0 \)

\[ n_1(0) = 7, n_2(0) = 0, n_3(0) = 0, n_4(0) = 0, \]

\[ n_5(0) = 1, n_6(0) = 1, n_7(0) = 1, n_8(0) = 0, \]

where

\[ n_1(0) + n_2(0) + n_3(0) + n_4(0) + n_5(0) + n_6(0) \]

\[ + n_7(0) + n_8(0) = 10; \]

- the vector of realizations of the numbers of the pipeline system operation process transitions in the particular operation states \( z_b \) at the initial moment \( t = 0 \)

\[ [n_b(0)] = [n_1(0), n_2(0), n_3(0), n_4(0), n_5(0), n_6(0)], \]

\[ = [7, 0, 0, 0, 1, 1, 0]; \]

- the realization \( n_{bl} \) of the numbers of pipeline system operation process transitions from the state \( z_b \) into the state \( z_l \) during the experiment time \( \Theta = 70 \text{ days} \)

\[ n_{b1} = 2, n_{b2} = 0, n_{b3} = 0, n_{b4} = 0, n_{b5} = 0, \]

\[ n_{b6} = 2, n_{b7} = 11, n_{b8} = 1, \]

\[ n_{b9} = 2, n_{b10} = 0, n_{b11} = 0, n_{b12} = 0, n_{b13} = 3, \]

\[ n_{b14} = 3, n_{b15} = 43, \]

\[ n_{b16} = 45, \]

\[ n_{b17} = 51, \]

\[ n_{b18} = 57; \]

\[ = \begin{bmatrix} n_{b1}, \ldots, n_{b8} \end{bmatrix}; \]

\[ = \begin{bmatrix} 0, 0, 0, 0, 3, 3, 43, 1 \end{bmatrix}; \]

\[ = \begin{bmatrix} 2, 0, 0, 0, 0, 2, 11, 1 \end{bmatrix}; \]

\[ = \begin{bmatrix} 0, 0, 0, 0, 9, 0, 0, 1 \end{bmatrix}; \]

\[ = \begin{bmatrix} 2, 0, 0, 0, 0, 0, 0, 1 \end{bmatrix}; \]

- the realization \( n_b \) of the total numbers of the pipeline system operation process transitions from the operation state \( z_b \) during the experiment time \( \Theta = 70 \text{ days} \) (the sums of the numbers of the matrix \([n_b]\])

\[ n_1 = n_{b1} + n_{b2} + n_{b3} + n_{b4} + n_{b5} + n_{b6} + n_{b7} + n_{b8} = 50, \]

\[ n_2 = n_{b9} + n_{b10} + n_{b11} + n_{b12} + n_{b13} + n_{b14} + n_{b15} + n_{b16} + n_{b17} + n_{b18} = 50; \]

\[ n_3 = n_{b19} + n_{b20} + n_{b21} + n_{b22} + n_{b23} + n_{b24} + n_{b25} + n_{b26} + n_{b27} + n_{b28} = 50, \]

\[ n_4 = n_{b29} + n_{b30} + n_{b31} + n_{b32} + n_{b33} + n_{b34} + n_{b35} + n_{b36} + n_{b37} + n_{b38} = 50, \]

\[ n_5 = n_{b39} + n_{b40} + n_{b41} + n_{b42} + n_{b43} + n_{b44} + n_{b45} + n_{b46} + n_{b47} + n_{b48} = 50. \]
Statistical identification and prediction of the port oil pipeline system's operation process and its reliability evaluation

5. The port oil pipeline system operation process characteristics evaluation

At the moment because of the luck of sufficient statistical data about the oil terminal operation process it is not possible to estimate its all operational characteristics. Namely, it is not possible to determine the matrix of the conditional distribution functions \( [H_{bl}(t)]_{b,l} \) of the lifetimes \( \theta_b^l \) for \( b, l = 1, 2, ..., 8, b \neq l \), (2) and further consequently, according to (20) [5], it is also not possible to determine the vector \( [H_b(t)]_{1,8} \) of the unconditional distribution functions of the lifetimes \( \theta_b^l \) of this operation process at the operation states \( z_b, b=1,2,...,8 \). However, on the basis of the preliminary statistical data coming from experiment it is possible to evaluate approximately the conditional mean values \( M_{bl} = E[\theta_b^l] \), \( b, l = 1, 2, ..., 8, b \neq l \), (3) of lifetimes in the particular operation states. On the basis of the statistical data given in Tables 1-10 in [4] their approximate evolutions are as follows:

\[
M_{15} = 720, M_{16} = 420, M_{17} = 698.95, M_{18} = 480, \\
M_{31} = 750, M_{36} = 564, M_{37} = 748.7, M_{38} = 540, \\
M_{41} = 360, M_{45} = 360, \\
M_{71} = 975.3, M_{75} = 872.4, M_{78} = 600, \\
M_{81} = 900, M_{47} = 420.
\]

Hence, by (5), the unconditional mean sojourn times in the particular operation states are given by:

\[
M_1 = E[\theta_1] = p_{15}M_{15} + p_{16}M_{16} + p_{17}M_{17} + p_{18}M_{18} = 0.06 \cdot 720 + 0.06 \cdot 420 + 0.86 \cdot 698.95 + 0.02 \cdot 480 \equiv 679.1, \\
M_4 = E[\theta_4] = p_{41}M_{41} + p_{45}M_{45} = 0.125 \cdot 750 + 0.125 \cdot 564 + 0.687 \cdot 748.7 + 0.025 \cdot 540 \equiv 712.63, \\
M_6 = E[\theta_6] = p_{61}M_{61} + p_{65}M_{65} = 0.4 \cdot 360 + 0.6 \cdot 360 = 360, \\
M_7 = E[\theta_7] = p_{71}M_{71} + p_{75}M_{75} + p_{78}M_{78}.
\]
\[ M_{s} = E[\theta_{s}] = p_{s1}M_{s1} + p_{s7}M_{s7} = 0.67 \cdot 900 + 0.33 \cdot 420 \equiv 741.6. \]

Since from the system of equations below (7)

\[
\begin{pmatrix}
\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8 \\
\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 = 1,
\end{pmatrix}
\]

we get

\[
\begin{align*}
\pi_1 &= 0.396, \quad \pi_2 = 0, \quad \pi_3 = 0, \quad \pi_4 = 0, \\
\pi_5 &= 0.116, \quad \pi_6 = 0.038, \quad \pi_7 = 0.435, \quad \pi_8 = 0.015.
\end{align*}
\]

Then the limit values of the transient probabilities \( p_{b}(t) \) at the operational states \( z_{b} \), according to (6), are given by

\[
\begin{align*}
p_1 &= 0.34, \quad p_2 = 0, \quad p_3 = 0, \quad p_4 = 0, \\
p_5 &= 0.1, \quad p_6 = 0.02, \quad p_7 = 0.53, \quad p_8 = 0.01.
\end{align*}
\]

6. Reliability of systems in variable operation process

We assume that the changes of the system operation process \( Z(t) \) states have an influence on the system components \( E_{i} \), \( i = 1, 2, \ldots, n \), reliability and the system reliability structure as well. Thus, we denote the conditional reliability function of the system component \( E_{i} \), while the system is at the operational state \( z_{b} \), \( b = 1, 2, \ldots, v \), by

\[
[R_{i}(t, \cdot)]^{(b)} = [1, [R_{i}(t, 1)]^{(b)} \ldots [R_{i}(t, z)]^{(b)}],
\]

for \( t \in -\infty, \), \( b = 1, 2, \ldots, v \), \( u = 1, 2, \ldots, z \), where

\[
[R_{i}(t, u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_{b})
\]

for \( t \in -\infty, \), \( i = 1, 2, \ldots, n \), \( b = 1, 2, \ldots, v \), \( u = 1, 2, \ldots, z \),

and the conditional reliability function of the system while the system is at the operational state \( z_{b} \), \( b = 1, 2, \ldots, v \), by

\[
[R_{n}(t, \cdot)]^{(b)} = [1, [R_{n}(t, 1)]^{(b)} \ldots [R_{n}(t, z)]^{(b)}],
\]

for \( t \in -\infty, \), \( b = 1, 2, \ldots, v \), \( u = 1, 2, \ldots, z \), where

\[
[R_{n}(t, u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_{b})
\]

for \( t \in -\infty, \), \( b = 1, 2, \ldots, v \), \( u = 1, 2, \ldots, z \), \( n \in N \),

and

\[
[R_{n}(t, u)]^{(b)} = R_{n}([R_{1}(t, u)]^{(b)}, [R_{2}(t, u)]^{(b)}, \ldots, [R_{n}(t, u)]^{(b)})
\]

for \( t \in -\infty, \), \( b = 1, 2, \ldots, v \), \( u = 1, 2, \ldots, z \), \( n \in N \).

The reliability function \( [R_{i}(t, u)]^{(b)} \) is the conditional probability that the component \( E_{i} \) lifetime \( T^{(b)}(u) \) in the reliability state subset \( \{u, u + 1, \ldots, z\} [5] \) is greater than \( t \), while the process \( Z(t) \) is at the operation state \( z_{b} \). Similarly, the reliability function \( [R_{n}(t, u)]^{(b)} \) is the conditional probability that the system lifetime \( T^{(b)}(u) \) in the reliability state subset \( \{u, u + 1, \ldots, z\} [5] \) is greater than \( t \), while the process \( Z(t) \) is at the operation state \( z_{b} \). In the case when the system operation time is large enough, the unconditional reliability function of the system is given by

\[
[R_{n}(t, \cdot)]^{(b)} = [1, [R_{n}(t, 1)]^{(b)} \ldots [R_{n}(t, z)]^{(b)}],
\]

where

\[
[R_{n}(t, u)]^{(b)} = P(T(u) > t) \equiv \sum_{b=1}^{v} p_{b} [R_{n}(t, u)]^{(b)}
\]

for \( t \in -\infty, \), \( u = 1, 2, \ldots, z \),

where \( T(u) \) is the unconditional lifetime of the system in the reliability state subset \( \{u, u + 1, \ldots, z\} \)

and the mean value of the system lifetime is
\( \mu(u) \equiv \sum_{a=1}^{\nu} p_a \mu_a(u), \quad u = 1, 2, \ldots, z, \)  \hspace{1cm} (11)

where

\[
\mu_b(u) = \int_0^\infty [R_b(t,u)]^{(b)} \, dt,
\]

(12)

and \( p_b \) are given by (6), and the variance of the system lifetime is

\[
\sigma^2(u) = 2 \int_0^\infty t \, R_b(t,u) \, dt - [\mu(u)]^2.
\]

(13)

7. Reliability and risk evaluation of port oil pipeline transportation system in variable operation process

After discussion with experts, taking into account the safety of the operation of the oil pipeline transportation system, we distinguish the following three reliability states of its components [2]:

- a reliability state 2 – piping operation is fully safe,
- a reliability state 1 – piping operation is less safe and more dangerous because of the possibility of environment pollution,
- a reliability state 0 – piping is destroyed.

Next, using the model considering in section 6, the results of section 5 and the results given in [4] by (9) and (10), we have

\[
R_n(t, u) = [1, R_n(t, 1), R_n(t, 2)],
\]

for \( t \geq 0, \)

\[
R_n(t, 2) = p_1[R_n(t, 2)]^{(1)} + p_2[R_n(t, 2)]^{(2)} + p_3[R_n(t, 2)]^{(3)} + p_4[R_n(t, 2)]^{(4)} + p_5[R_n(t, 2)]^{(5)} + p_6[R_n(t, 2)]^{(6)} + p_7[R_n(t, 2)]^{(7)} + p_8[R_n(t, 2)]^{(8)}
\]

(15)

where \([R_n(t, 1)]^{(1)}, [R_n(t, 1)]^{(2)}, [R_n(t, 1)]^{(3)}, [R_n(t, 1)]^{(4)}, [R_n(t, 1)]^{(5)}, [R_n(t, 1)]^{(6)}, [R_n(t, 1)]^{(7)}, [R_n(t, 1)]^{(8)}\) and \([R_n(t, 2)]^{(1)}, [R_n(t, 2)]^{(2)}, [R_n(t, 2)]^{(3)}, [R_n(t, 2)]^{(4)}, [R_n(t, 2)]^{(5)}, [R_n(t, 2)]^{(6)}, [R_n(t, 2)]^{(7)}, [R_n(t, 2)]^{(8)}\) are the system reliability functions in particular operation states determined by (23), (29), (41), (56), (69), (84), (99), (105) and (24), (30), (42), (57), (70), (85), (100), (106) given in [4]. Since according to the results given in [4], the mean values of the conditional system lifetimes in the reliability state subsets by (12) in years are:

\[
\mu_1(1) \equiv 0.364, \quad \mu_1(2) \equiv 0.304,
\]

\[
\mu_2(1) \equiv 0.807, \quad \mu_2(2) \equiv 0.666,
\]

\[
\mu_3(1) \equiv 0.307, \quad \mu_3(2) \equiv 0.218,
\]

\[
\mu_4(1) \equiv 0.079, \quad \mu_4(2) \equiv 0.058,
\]

\[
\mu_5(1) \equiv 0.307, \quad \mu_5(2) \equiv 0.218,
\]

\[
\mu_6(1) \equiv 0.079, \quad \mu_6(2) \equiv 0.058,
\]

\[
\mu_7(1) \equiv 0.11, \quad \mu_7(2) \equiv 0.083,
\]

\[
\mu_8(1) \equiv 0.364, \quad \mu_8(2) \equiv 0.304,
\]
then applying (14), (15), (11) and (13), we get the mean value and the standard deviation of the system unconditional lifetime in the reliability state subsets given by

\[
\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1) + p_4\mu_4(1) \\
+ p_5\mu_5(1) + p_6\mu_6(1) + p_7\mu_7(1) + p_8\mu_8(1) \\
\cong 0.218, \\
\sigma(1) \cong 0.228 \text{ year},
\]

\[
\mu(2) = p_1\mu_1(2) + p_2\mu_2(2) + p_3\mu_3(2) + p_4\mu_4(2) \\
+ p_5\mu_5(2) + p_6\mu_6(2) + p_7\mu_7(2) + p_8\mu_8(2) \\
\cong 0.173, \\
\sigma(2) \cong 0.185 \text{ year}.
\]

If the critical safety state [2] is \( r = 1 \), then the system risk function, is given by

\[
r(t) = 1 - R(t, 1) \\
= 1 - [0.34 \cdot \overline{R}(t, 1)]^{10} + 0.1 \cdot \overline{R}(t, 1)^3 \\
+ 0.02 \cdot \overline{R}(t, 1)^6 + 0.53 \cdot \overline{R}(t, 1)^7 \\
+ 0.01 \cdot \overline{R}(t, 1)^{10} \text{ for } t \geq 0.
\]

Hence, the moment when the system risk function exceeds a permitted level, for instance \( \delta = 0.05 \), is

\[
\tau = r^{-1}(\delta) \cong 0.011 \text{ years}.
\]

8. Conclusion

The paper proposes an approach to the solution of practically very important problem of linking the systems’ reliability and their operation processes. To involve the interactions between the systems’ operation processes and their varying in time reliability structures and components’ reliability characteristics a semi-markov model of the systems’ operation processes and system conditional reliability functions are used. This approach gives practically important in everyday usage tool for reliability evaluation of the systems with changing reliability structures and components’ reliability characteristics during their operation processes.

Application of the proposed method is illustrated in the reliability and risk evaluation of the port oil pipeline transportation system. The reliability input data concerned with the operation process and reliability functions of the components of the port oil transportation system are not precise. They are coming from experts and are concerned with the mean lifetimes of the system components and with the conditional sojourn times of the system in the operation states. To improve the achieved results it is supposed that the statistical data given in [4] will be collected for the next two years and after this period of time the full identification of the pipeline oil transportation operation process will be performed and this process main characteristics will be determined and used in pipeline transportation systems reliability, risk and availability more precise analysis and evaluation.

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