1. Introduction

Complex technical systems improvement and especially standby redundancy are basic and fundamental concept used in the process of system designing and in the reliability analysis [12]. Although the method of reliability analysis of improved systems are limited, there are still appearing new works on this topic. For example in [1] there is proposed a new method of reliability analysis of systems with cold-warm-hot standby components and shared standby component among different subsystems, operation at different conditions, using the concept of counting processes. Standby redundancy is a technique that has been widely applied to improving and optimize system reliability and availability in system design [13]. As an example there can be quoted a modern terminal computer system of remote control and supervisory device of electric system power generator. This system is equipped in hot reservation devices. In [5] a cold standby redundancy is used as a method to determine an optimal design configuration for non-repairable series-parallel systems that maximize system reliability.

In the paper there is presented the computer program based on the results of [6] including the methods of the complex technical systems with reserved and improved components reliability. The computer program allows for automatic prediction of improved complex technical systems reliability using these methods: a hot and a cold single reservation of system components and replacing the system components by improved components with reduced intensities of departure from the reliability state subsets [8, 11]. The reliability characteristics can be found for the improved this way series, parallel, “m out of n”, consecutive “m out of n: F”-series, parallel-series, series-“m out of n”, “m out of n”-series, series-consecutive “m out of n: F” and consecutive “m out of n: F”-series complex technical systems and for the series systems composed these systems [9], [10].

2. Description of the computer program

The computer program is written in Java language using SSJ V2.1.3 library. The SSJ library is a Java library, developed in the Department d’Informatique et de Recherche Operationelle (DIRO) at the
Universite de Montreal, that gives the support of stochastic simulations. The on-line documentation of SSJ can be found at the website http://www.iro.umontreal.ca/~simardr/ssj/indexe.htm. The scheme of the computer program is presented in Figure 1.

Figure 1. The scheme of the algorithm of the computer program for prediction of improved complex technical systems reliability and safety
2.1. Input parameters

The computer program is composed of three panels. The first panel “Parameters of the system operation process” is used for reading input parameters of the system operation process:
- the number of operation states of the system operation process \( n \),
- the transient probabilities in particular operation states \( p_1, p_2, \ldots, p_n \).

The second panel “Reliability parameters of the system” is served for reading input parameters of the system reliability model and method of improvement:
- the number of the system and components reliability states \( z + 1 \),
- the system and components critical state \( r \),
- the system risk permitted level \( \delta \),
- the method of system reliability improvement:
  - a hot single reservation of system components,
  - a cold single reservation of system components,
  - replacing the system components by improved components with reduced intensities of departure from the reliability state subsets,
- the parameters of a system reliability structure in various operation states,
- the intensities of components departure (equivalently \( [\lambda_i(u)]^{(b)} \) or \( [\lambda_j(u)]^{(b)} \)) from the reliability states subset \( \{u, u + 1, \ldots, z\} \), \( u = 1, 2, \ldots, z \), at the operation state \( z_b \), \( b = 1, 2, \ldots, \nu \), assuming that the reliability functions of the system components are exponential,
- the factor \( \rho^{(b)}(u) \), \( 0 < \rho^{(b)}(u) \leq 1 \) (equivalently factors \( \rho^{(b)}_i(u) \) or \( \rho^{(b)}_j(u) \)), in a case of replacing the system components by improved components with reduced intensities of departure from the reliability state subsets \( \{u, u + 1, \ldots, z\} \), \( u = 1, 2, \ldots, z \), at the operation state \( z_b \), \( b = 1, 2, \ldots, \nu \), by multiplying intensities by this factor.

2.2. Results – reliability characteristics

In the third panel “Characteristics of the system reliability and risk” there are presented reliability and risk characteristics of the system before and after system improvement [1].
There are determined the following characteristics:
- the unconditional reliability function of the system (with plotting) before
  \[
  R(t, \cdot) = [R^{(1)}(t, 1), \ldots, R^{(k)}(t, z)],
  \]
  and after the system improvement
  \[
  R^{(k)}(t, \cdot) = [1, R^{(k)}(t, 1), \ldots, R^{(k)}(t, z)], \ k \in \{1,2,3\},
  \]
  where \( R^{(k)}(t, \cdot) \) denotes the improved unconditional reliability function of the system with a hot single reservation of its components, \( R^{(k)}(t, \cdot) \) denotes the improved unconditional reliability function of the system with a cold single reservation of its components and \( R^{(3)}(t, \cdot) \) denotes the improved unconditional reliability function of the system with reduced intensities of its components departure,
- the mean values of the system conditional lifetimes in the reliability state subsets \( \{u, u + 1, \ldots, z\} \) while the system is at the operational state \( z_b \), \( b = 1, 2, \ldots, \nu \), before the system improvement
  \[
  \mu_b(u) = \frac{\infty}{0} [R(t, u)]^{(b)} dt, \ u = 1, 2, \ldots, z,
  \]
  and after the system improvement
  \[
  \mu^{(k)}_b(u), \ u = 1, 2, \ldots, z, \ k \in \{1,2,3\},
  \]
- the mean values of the system unconditional lifetimes in the reliability state subsets \( \{u, u + 1, \ldots, z\} \) before the system improvement
  \[
  \mu(u), \ u = 1, 2, \ldots, z,
  \]
  and after the system improvement
  \[
  \mu^{(k)}(u), \ u = 1, 2, \ldots, z, \ k \in \{1,2,3\},
  \]
- the standard deviations of the system unconditional lifetimes in the reliability state subsets \( \{u, u + 1, \ldots, z\} \) before the system improvement
  \[
  \sigma(u), \ u = 1, 2, \ldots, z,
  \]
  and after the system improvement
  \[
  \sigma^{(k)}(u), \ u = 1, 2, \ldots, z, \ k \in \{1,2,3\},
  \]
- the mean values of the system unconditional lifetimes in the particular reliability states before the system improvement
  \[
  \overline{R}(u), \ u = 1, 2, \ldots, z,
  \]
  and after the system improvement
  \[
  \overline{R}^{(k)}(u), \ u = 1, 2, \ldots, z, \ k \in \{1,2,3\},
  \]
- the system risk function (with plotting) before the system improvement
  \[
  \begin{align*}
  &\overline{K}(u, \cdot), \ u = 1, 2, \ldots, z, \\
  &\overline{K}^{(k)}(u, \cdot), \ u = 1, 2, \ldots, z, \ k \in \{1,2,3\},
  \end{align*}
  \]
and after the system improvement

\[ r^{(k)}(t), \quad t \in <0,\infty), \]

– the moment when the risk exceeds a permitted level \( \delta \) before the system improvement \( \tau \),

and after system improvement \( \tau^{(k)}, k \in \{1,2,3\} \).

3. Application

3.1. The exemplary system analysis

We analyze the reliability of an exemplary system \( S \) that consists of two subsystems \( S_1, S_2 \). The subsystem \( S_1 \) is a series system composed of 4 components, denoted respectively by

\[ E_1^{(i)}, \quad i = 1,2,3,4, \]

with the reliability structure presented in Figure 2.

\[ E_1^{(1)} \rightarrow E_2^{(1)} \rightarrow E_3^{(1)} \rightarrow E_4^{(1)} \]

Figure 2. The scheme of the system \( S_1 \) reliability structure

The subsystem \( S_2 \) is a parallel system composed of 2 components, denoted respectively by

\[ E_1^{(2)}, \quad i = 1,2, \]

with the reliability structure presented in Figure 3.

\[ E_1^{(2)} \rightarrow E_2^{(2)} \]

Figure 3. The scheme of the system \( S_2 \) reliability structure

The subsystems \( S_1, S_2 \), illustrated in Figures 2–3 are forming a series reliability structure presented in Figure 4.

3.2. The exemplary system operation process

Under the assumption that the exemplary system structure and the subsystem components reliability depend on its changing in time operation states, we arbitrarily fix the number of the system operation process states \( \nu = 3 \) and we distinguish the following operation states:

- an operation state \( z_1 \) – the system is composed of the subsystem \( S_1 \), with the scheme showed in Figure 2, that is a series system,

- an operation state \( z_2 \) – the system is composed of the subsystem \( S_2 \), with the scheme showed in Figure 3 that is a parallel system,

- an operation state \( z_3 \) – the system is composed of the subsystems \( S_1 \) and \( S_2 \), with the scheme showed in Figure 4, while the subsystem \( S_1 \) is a series system and the subsystem \( S_2 \) is a parallel system (Figure 5).

\[ E_1^{(2)} \rightarrow E_2^{(2)} \rightarrow E_3^{(2)} \rightarrow E_4^{(2)} \]

Figure 5. The scheme of the system reliability structure at operation state \( z_3 \)

We arbitrarily assume that the transient probabilities of the system at particular operation states \( z_1, z_2, z_3 \), respectively are

\[ p_1 = 0.5, \quad p_2 = 0.25, \quad p_3 = 0.25. \]

3.3. The exemplary system components reliability

We assume that the exemplary system and its components have four reliability states 0, 1, 2, 3, i.e. \( z = 3 \). And consequently, at all operation states \( z_b, b = 1,2,3 \), we arbitrarily distinguish the following reliability states of the system and its components:

- a reliability state 3 – the system operation is fully effective,

- a reliability state 2 – the system operation is less effective because of ageing,
• a reliability state 1 – the system operation is less effective because of ageing and more dangerous,
• a reliability state 0 – the system is destroyed.
We assume that the changes of the system operation process states have an influence on changing the system multi-state components reliability and the system reliability structure as well. We assume that the system critical reliability state is \( r = 2 \).
Consequently, we define the four-state conditional reliability functions of the system components \( E_i^{(v)} \), \( v = 1,2, \)
\[
[R_i^{(v)}(t, \cdot)]^{(b)} = \exp\{-\lambda_i^{(v)}(t) + \lambda_i^{(v)}(t, 1)\},
\]
\[
[R_i^{(v)}(t, 3)]^{(b)} = \exp\{-\lambda_i^{(v)}(t) + \lambda_i^{(v)}(t, 2)\},
\]
with the exponential co-ordinates
\[
[R_i^{(v)}(t, 1)]^{(b)} = \exp\{-\lambda_i^{(v)}(1)\},
\]
\[
[R_i^{(v)}(t, 2)]^{(b)} = \exp\{-\lambda_i^{(v)}(2)\},
\]
\[
[R_i^{(v)}(t, 3)]^{(b)} = \exp\{-\lambda_i^{(v)}(3)\},
\]
different in various operation states \( z_b \), \( b = 1,2,3, \) where \( \lambda_i^{(v)}(1) \), \( \lambda_i^{(v)}(2) \), \( \lambda_i^{(v)}(3) \), \( b = 1,2,3, \)
\( v = 1,2, \) are the subsystems components unknown intensities of departures respectively from the reliability state subsets \( \{1,2,3\} \).

At the system operation state \( z_2 \), the system is composed of the series subsystem \( S_2 \) composed of four components \( n = 4 \) with the reliability structure showed in Figure 2.
In the subsystem \( S_2 \) there are:

- the component \( E_2^{(1)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_2^{(1)}(1)]^{(1)} = 0.1, \quad [\lambda_2^{(1)}(2)]^{(1)} = 0.2,
\]
\[
[\lambda_2^{(1)}(3)]^{(1)} = 0.3;
\]

- the component \( E_2^{(2)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_2^{(2)}(1)]^{(2)} = 0.1, \quad [\lambda_2^{(2)}(2)]^{(2)} = 0.23,
\]
\[
[\lambda_2^{(2)}(3)]^{(2)} = 0.35;
\]

- the component \( E_2^{(3)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_2^{(3)}(1)]^{(3)} = 0.1, \quad [\lambda_2^{(3)}(2)]^{(3)} = 0.2, \quad [\lambda_2^{(3)}(3)]^{(3)} = 0.3;
\]

At the system operation state \( z_3 \), the system is composed of subsystems \( S_1 \) and \( S_2 \) linked in series with the scheme showed in Figure 3.
In the subsystem \( S_2 \) there are:

- the component \( E_1^{(1)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_1^{(1)}(1)]^{(1)} = 0.15, \quad [\lambda_1^{(1)}(2)]^{(1)} = 0.2,
\]
\[
[\lambda_1^{(1)}(3)]^{(1)} = 0.35;
\]

- the component \( E_2^{(1)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_2^{(1)}(1)]^{(2)} = 0.15, \quad [\lambda_2^{(1)}(2)]^{(2)} = 0.25,
\]
\[
[\lambda_2^{(1)}(3)]^{(2)} = 0.35.
\]

At the system operation state \( z_2 \), the system is composed of the parallel subsystem \( S_2 \) composed of two components \( n = 2 \) with the reliability structure showed in Figure 3.
In the subsystem \( S_2 \) there are:

- the component \( E_2^{(2)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_2^{(2)}(1)]^{(2)} = 0.2, \quad [\lambda_2^{(2)}(2)]^{(2)} = 0.4,
\]
\[
[\lambda_2^{(2)}(3)]^{(2)} = 0.6;
\]

- the component \( E_2^{(3)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_2^{(3)}(1)]^{(3)} = 0.2, \quad [\lambda_2^{(3)}(2)]^{(3)} = 0.46,
\]
\[
[\lambda_2^{(3)}(3)]^{(3)} = 0.7.
\]
At the system operation state \( z_3 \), the system is composed of subsystems \( S_1 \) and \( S_2 \) linked in series with the scheme showed in Figure 4, 5.
In the first subsystem \( S_1 \), that is a series system, there are:

- the component \( E_1^{(1)} \) with the intensities of departure from the reliability states subsets \( \{1,2,3\} \), \( \{2,3\} \), \( \{3\} \), respectively

\[
[\lambda_1^{(1)}(1)]^{(1)} = 0.1, \quad [\lambda_1^{(1)}(2)]^{(1)} = 0.2, \quad [\lambda_1^{(1)}(3)]^{(1)} = 0.3;
\]
− the component $E_2^{(1)}$ with the intensities of departure from the reliability states subsets \{1,2,3\}, \{2,3\}, \{3\}, respectively  
\[
[λ_2^{(1)}(1)]^{(3)} = 0.1, [λ_2^{(1)}(2)]^{(3)} = 0.23,  
[λ_2^{(1)}(3)]^{(3)} = 0.35;  
\]

− the component $E_3^{(1)}$ with the intensities of departure from the reliability states subsets \{1,2,3\}, \{2,3\}, \{3\}, respectively  
\[
[λ_3^{(1)}(1)]^{(3)} = 0.15, [λ_3^{(1)}(2)]^{(3)} = 0.2,  
[λ_3^{(1)}(3)]^{(3)} = 0.35;  
\]

− the component $E_4^{(1)}$ with the intensities of departure from the reliability states subsets \{1,2,3\}, \{2,3\}, \{3\}, respectively  
\[
[λ_4^{(1)}(1)]^{(3)} = 0.15, [λ_4^{(1)}(2)]^{(3)} = 0.25,  
[λ_4^{(1)}(3)]^{(3)} = 0.35.  
\]

In the second subsystem $S_2$, that is a parallel system, there are:

− the component $E_1^{(2)}$ with the intensities of departure from the reliability states subsets \{1,2,3\}, \{2,3\}, \{3\}, respectively  
\[
[λ_1^{(2)}(1)]^{(3)} = 0.2, [λ_1^{(2)}(2)]^{(3)} = 0.4,  
[λ_1^{(2)}(3)]^{(3)} = 0.6;  
\]

− the component $E_2^{(2)}$ with the intensities of departure from the reliability states subsets \{1,2,3\}, \{2,3\}, \{3\}, respectively  
\[
[λ_2^{(2)}(1)]^{(3)} = 0.2, [λ_2^{(2)}(2)]^{(3)} = 0.46,  
[λ_2^{(2)}(3)]^{(3)} = 0.7.  
\]

3.4. The exemplary system reliability improvement

In practice, to improve the reliability and safety of the complex systems, besides of the optimization of their operation processes their qualitative and quantitative redundancy is also used. The most popular ways of the system reliability and safety improvement are using the hot and cold reservations of the basic system components and replacing the basic components by the improved components with reduced intensities of departure from the reliability state subsets. These methods are also used to improve the reliability of the considered exemplary system.

We assume that the reserve components are identical with the basic components in reliability sense, i.e. they have the same multi-state exponential reliability functions with the intensities $[λ_i^{(v)}(1)]^{(b)}$, $[λ_i^{(v)}(2)]^{(b)}$, $[λ_i^{(b)}(3)]^{(b)}$, $v=1,2$, of departure from the reliability state subsets \{1,2,3\}, \{2,3\}, \{3\}, at the operation state $z_b$, $b=1,2,3$.

Selecting the method of replacing the system components by the improved components with reduced intensities of departure from the reliability state subsets it is necessary to fix factors $[ρ_i^{(v)}(u)]^{(b)}$, $v=1,2$, $u=1,2,3$, $b=1,2,3$.

At the system operation state $z_1$, in the subsystem $S_1$ we assume that only components $E_1^{(3)}$ and $E_2^{(3)}$ are improved by multiplying their intensities of departure by factor $[ρ_i^{(v)}(u)]^{(b)}$, $i=3,4$, $u=1,2,3$:

− for the component $E_1^{(1)}$ the factors are equal
  
  $[ρ_1^{(1)}(1)]^{(1)} = 1$, $[ρ_1^{(1)}(2)]^{(1)} = 1$, $[ρ_1^{(1)}(3)]^{(1)} = 1$;

− for the component $E_2^{(1)}$ the factors are equal
  
  $[ρ_2^{(1)}(1)]^{(1)} = 1$, $[ρ_2^{(1)}(2)]^{(1)} = 1$, $[ρ_2^{(1)}(3)]^{(1)} = 1$;

− for the component $E_3^{(1)}$ the factors are equal
  
  $[ρ_3^{(1)}(1)]^{(1)} = 0.8$, $[ρ_3^{(1)}(2)]^{(1)} = 0.5$, $[ρ_3^{(1)}(3)]^{(1)} = 0.5$;

− for the component $E_4^{(1)}$ the factors are equal
  
  $[ρ_4^{(1)}(1)]^{(1)} = 0.8$, $[ρ_4^{(1)}(2)]^{(1)} = 0.5$, $[ρ_4^{(1)}(3)]^{(1)} = 0.5$.  

Figure 6. Selection of the method of the system reliability improvement in the program
At the system operation state $z_2$, in the subsystem $S_2$ we assume that components $E_i^{(2)}$ and $E_j^{(2)}$ are improved by multiplying their intensities of departure by factor $\rho_{i}^{(2)}(u)$, $i = 1, 2, u = 1, 2, 3$:

- for the component $E_i^{(2)}$ the factors are equal

$$[\rho_{i}^{(2)}(1)]^{(2)} = 0.75, \ [\rho_{i}^{(2)}(2)]^{(2)} = 0.5,$$

$$[\rho_{i}^{(2)}(3)]^{(2)} = 0.5;$$

- for the component $E_j^{(2)}$ the factors are equal

$$[\rho_{j}^{(2)}(1)]^{(2)} = 0.75, \ [\rho_{j}^{(2)}(2)]^{(2)} = 0.5,$$

$$[\rho_{j}^{(2)}(3)]^{(2)} = 0.5.$$

At the system operation state $z_3$, in the subsystem $S_1$ we assume that only components $E_3^{(1)}$ and $E_4^{(1)}$ are improved by multiplying their intensities of departure by factor $\rho_{i}^{(1)}(u)$, $i = 3, 4, u = 1, 2, 3$:

- for the component $E_1^{(1)}$ the factors are equal

$$[\rho_{1}^{(1)}(1)]^{(1)} = 1, \ [\rho_{1}^{(1)}(2)]^{(1)} = 1, \ [\rho_{1}^{(1)}(3)]^{(1)} = 1;$$

- for the component $E_2^{(1)}$ the factors are equal

$$[\rho_{2}^{(1)}(1)]^{(1)} = 1, \ [\rho_{2}^{(1)}(2)]^{(1)} = 1, \ [\rho_{2}^{(1)}(3)]^{(1)} = 1;$$

- for the component $E_3^{(1)}$ the factors are equal

$$[\rho_{3}^{(1)}(1)]^{(1)} = 0.8, \ [\rho_{3}^{(1)}(2)]^{(1)} = 0.5,$$

$$[\rho_{3}^{(1)}(3)]^{(1)} = 0.5;$$

- for the component $E_4^{(1)}$ the factors are equal

$$[\rho_{4}^{(1)}(1)]^{(1)} = 0.8, \ [\rho_{4}^{(1)}(2)]^{(1)} = 0.5,$$

$$[\rho_{4}^{(1)}(3)]^{(1)} = 0.5.$$

---

**Figure 7.** The results of the computer program for the exemplary system with a hot reservation
3.4.1. Reliability and risk characteristics of the system with hot reservation

For the exemplary system with a hot single reservation of its components the obtained reliability and risk characteristics, presented in Point 2.2, are given in the widows below. In the first window there are given the reliability characteristics of the considered exemplary system before and in the second window after the system improvement. The program gives possibility of showing the plot of the coordinates of the system unconditional reliability function in the reliability state subset by pressing the button “Reliability Plot” for both widows i.e. for the system before and after improvement. Then the following window with the plot appears on the screen.

It is also possible to obtain the plot of the system risk function with marked moment when the risk exceeds a permitted level by pressing the button “Risk Plot”.

![Figure 8](image)

**Figure 8.** The plot of the unconditional reliability function coordinates of the exemplary system with a hot reservation

3.4.2. Reliability and risk characteristics of the system with cold reservation

For the system with a cold single reservation of its components we obtain the reliability and risk characteristics mentioned in Point 2.2. Below in the computer window there are presented results for the considered exemplary system.

![Figure 9](image)

**Figure 9.** The plot of the exemplary system risk function with a hot reservation
Figure 10. The results of the computer program for the exemplary system with a cold reservation

Figure 11. The plot of the unconditional reliability function coordinates of the exemplary system with a cold reservation

3.4.3. Reliability and risk characteristics of the system with improved components

The results of the computer program for prediction of improved system reliability for the exemplary system with reduced intensities of departure of its components are presented below.
Figure 12. The results of the computer program for the exemplary system with reduced intensities of departure

Figure 13. The plot of the unconditional reliability function coordinates of the exemplary system before improvement

4. Conclusion

Redundancy is a common approach to improve the reliability and availability of a system [12]. Thus presented in the paper computer program allowing for automatic prediction of improved complex technical systems can be used as a helpful practical tool in real systems design and resources and efforts of system improvement while the system’s performance [2].
The presented computer program for prediction of improved complex technical systems reliability and safety is based on methods and algorithms presented in [3]. The computer program determines the reliability characteristics of the improved complex technical systems with hot and cold single reservation of their components and of the improved complex technical systems with reduced intensities of departure from the reliability state subsets of their components under the assumption that the considered systems are exponential.

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